SUPERCONDUCTIVITY

Superconductivity was first observed in 1911 by the Dutch Physicist H. Kamerlingh Onnes when he was studying the conductivity of metals at low temperatures. He found that resistivity of pure mercury metal Vanishes abruptly at 4.2K. The temperature at which this transition happens is called critical Temperature T_c . This effect of vanishing of resistivity below a critical temperature is called superconductivity.

The Superconducting transition is reversible. Above the critical temperature the metals is in a normal state while below T_0 . The Superconducting state is found to be a new state of mater.

Resistivity of a metals is given by equation

$$\rho = \frac{m}{ne^2 \tau}$$

m → mass of electron, n → the concentration and τ → the collision time.

As temperature decreases the lattice vibrations begin to decrease this results in longer collision time and hence ρ decreases. At extremely low temperatures $\tau \rightarrow \infty$ resulting in $\rho \rightarrow 0$ and hence electrons undergo no scattering.

For a pure metals which are structurally perfect, the super conducting transition happens at a sharp temperature. For an impure metal, the superconducting transition temperature is not sharp.

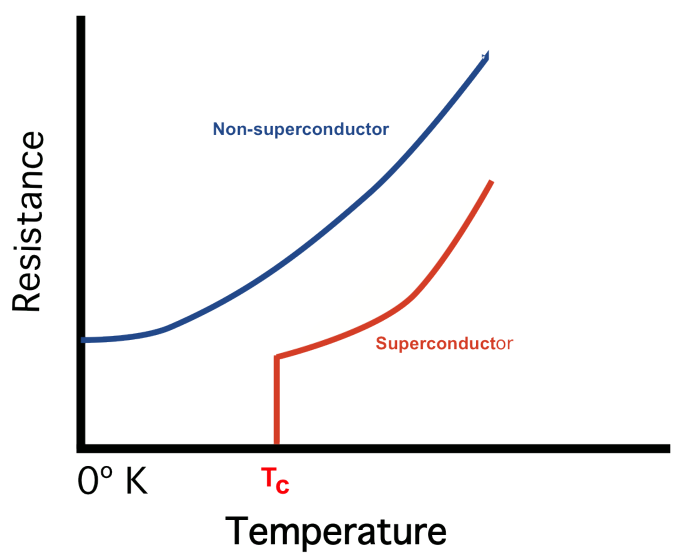

Fig-1

Empirical Facts

1) Superconductivity occurs only in substances where the valence electrons is between 2 and 8.
   The phenomenon is not observed generally in alkali or noble metals.
2) A small atomic volume accompanied by a small atomic man favours superconductivity.
3) Substances with odd number of valence electrons and favourable for super conductivity while with even number of valence electrons and unfavorable.

Perfect diamagnetism or Meissner effect

It is observed that super conductor expels magnetic flame completely and the phenomenon is called Meissner – ochsenfeld effect.

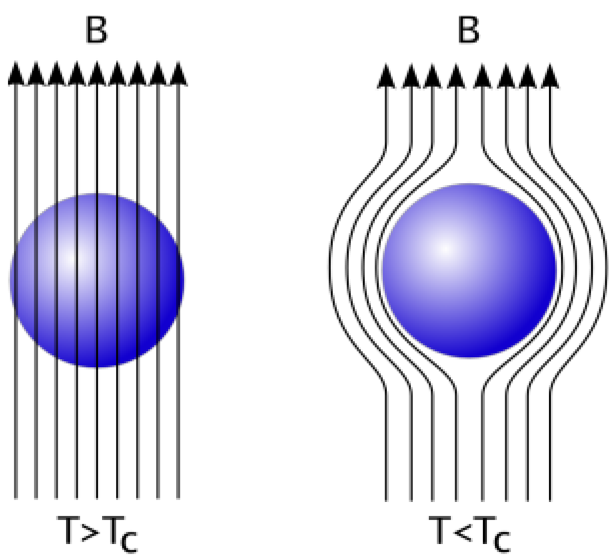

Fig-2

As soon as the temperature is lowered to critical temperature T_c the metal becomes superconducting and all the magnetic lines of force are expelled. It is also seen that the effect is reversible.

The magnetic induction inside a substance is given by

$$B=\mu_0(H+M)$$

Where H is the intensity of the magnetic field and M → the magnetization in the medium

$$B = \mu_0 H \left[1 + \frac{M}{H}\right]$$

$\frac{M}{H} = \chi_m$, the magnetic susceptibility

ie $B = \mu_0 H (1 + \chi_m)$

For a super conductor magnetic induction is expelled hence B = 0

ie $\mu_0 H (1 + \chi_m) = 0$

ie $1 + \chi_m = 0$ or $\chi = 1$

ie $\frac{M}{H} = -1$

ie the magnetization cancels external intensity exactly. This phenomenon is called diamagnetism. Then a super conductor behaves as a perfect diamagnet.

NPTEL-(https://www.youtube.com/watch?v=GglT1RoBPzg)

Critical Field

Super conductivity can be destroyed by the application of magnetic field. For a particular value of applied magnetic field called critical field (Hc) a superconductor becomes a normal metal.

The critical field Hc depends on temperature. For a given substance, the field decreases as the temperature rises from T = 0 k to T =Tc

Experimentally it has been found that

$$H_c(T) = H_c(O)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

When T = Ok $H_c(T) = H_c(O)$

At T= Tc $H_c(T) = 0$ ie normal state

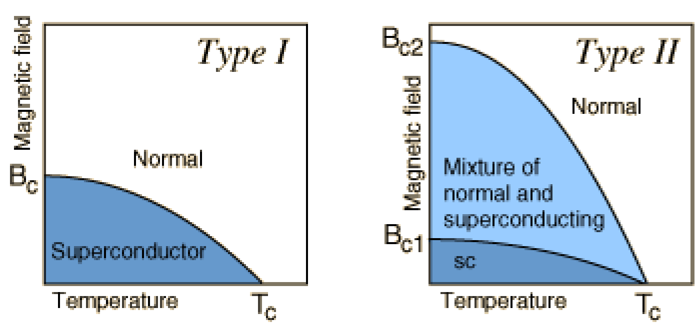

Fig-3

Electrodynamics of Super conductors

The electrodynamic properties were put forward by F. London in 1950. It explains known observations about superconductors.

It uses two fluid model put forward by Gorter and Casimir. According to this model the electrons in a super conductor are of two types – super-electrons and normal electrons. The normal electrons behave normally as in a metal at room temperature. But super electrons exhibit certain new properties so that it becomes super conducting. There electrons experience no scattering, have zero entropy and a long coherence length or spatial extension. The no. of super electrons varies with temperature as

$$n_s = n_0\left[1-\left(\frac{T}{T_c}\right)^4\right]$$

London's equations

At Ok superconductor contains only super electrons and at transition temperature all electrons are normal.

At any other temperature below Tc

$n = n_s + n_n$ $\quad n_s \rightarrow$ density of super e-

n_n = density of normal e-

Let j be the current density. n = density of electrons

$\therefore\ j = J_s + J_n$

Where J_s is the superconducting current density and J_n current density due to normal electrons.

But $J_n = e\ n_n\ V_n$ and $J_s = e\ n_s\ V_s$ where μ_n is the velocity of normal electrons and μ_S the velocity of super electrons.

The equation of motion of a superconducting electrons is

$$\frac{md\mu_s}{dt} = eE \qquad \text{..........(1)}$$

Where E is the externally applied electric field.

But $J_s = en_s\mu_s$ ....(2)

$$\frac{dJ_s}{dt} = en_s\frac{d\mu_s}{dt} \qquad \text{..........(3)}$$

ie $J_s = e n_s \mu_s$ ....(4)

From eqn. (1) $\mu_s = \frac{eE}{m}$

Sub : (5) in (4) we get

$$J_s = e n_s \frac{eE}{m} = \frac{n_s e^2 E}{m} \qquad \text{.............(6)}$$

For a steady current $J_s = 0$

ie J_s = a constant

ie E = 0 ....(7)

for a super conductor ie the electric field inside a superconductor vanishes

For a normal metal

$$J_n = \sigma_n E = \frac{n_n e^2 \tau E}{m} \qquad \text{.............(8)}$$

If E = 0 Jn = 0 ....(9)

Ie By only applying a voltage a current flows.

From Maxwell's equation

$$\nabla \times E = \frac{-\partial B}{\partial t} \qquad \text{............(10)}$$

If E = 0 as in the case of a super conductor.

$\frac{\partial B}{\partial t} = 0$ ie B = a constant

But Meissner effect tells that B = 0

Inside a super conductor.

For steady currents Maxwell's equation is

$$\nabla \times B = \mu_0 j \qquad \text{...............(12)}$$

and $\nabla \times E = -\frac{\partial B}{\partial t}$ ....(13)

ie Curl $B = \mu_0 J_s$ ....(14)

Curl $E = -B c^o$ ....(15)

But $J_s = \frac{n_s e^2 E}{m}$ ....(16)

$$E = \frac{m j_s}{n_s e^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

$$\therefore \overset{o}{B} = -Curl\ E = -curl \frac{m j_s}{n_s e^2} \qquad \ldots\ldots\ldots\ldots\ldots(18)$$

$$\overset{o}{B} = -\frac{m}{n_s e^2}\ curl\ j_s \qquad \ldots\ldots\ldots\ldots\ldots(19)$$

Also curl $B = \mu_0\ J_s$ ……………(20)

Differentiating both sides

$$\frac{d}{dt} curl\ b = \mu_0\ j_s \qquad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

ie curl $\overset{o}{B} = \mu_0\ j_s$ ……………(22)

Taking curl on both sides

Curl Curl $\overset{o}{B} = \mu_0\ curl\ \ j_s$ ……………(23)

Ie curl $j_s = \frac{1}{\mu_0}\ curl\ curl\ \overset{o}{B}$ ……………(24)

But $\overset{o}{B} = \frac{-m}{n_s e^2}\ curl\ j_s$ ……………(25)

Curl $j_s = \frac{-n_s e^2}{m} \overset{o}{B}$ ……………(26)

Substituting back in equation (24)

$$\frac{-n_s e^2}{m} \overset{o}{B} = \frac{1}{\mu_0}\ curl\ curl\ \overset{o}{B} \qquad \ldots\ldots\ldots\ldots(27)$$

ie $\overset{o}{B} = \frac{-m}{\mu_0 n_s e^2}\ curl\ curl\ \overset{o}{B}$ …………(28)

Let $\alpha = \frac{m}{\mu_0 n_s e^2}$ ……………(29)

ie $\overset{o}{B} = -\alpha\ \ curl\ \ curl\ \overset{o}{B}$ ……………(30)

From vector calculus

$$\nabla\times(\nabla\times A)=\nabla(\nabla . A)-\nabla^2 A$$

ie curl curl A = grad div $A-\nabla^2 B$ ....(31)

Invoking this identity

Curl Curl $\overset{o}{B}$ = grad div $\overset{o}{B}-\nabla^2\overset{o}{B}$ ....(32)

But div $\overset{o}{B}=0$ ....(33) (Maxwell's eqn)

$$\nabla . B=0 \qquad \nabla .\frac{\partial B}{\partial t}=0$$

$$\frac{d}{dt}(\nabla . B)=0$$

Equation (3) becomes

$$\overset{o}{B}=\alpha\nabla^2\overset{o}{B}$$

ie $\nabla^2\overset{o}{B}=\frac{\overset{o}{B}}{\alpha}$ ....(34)

But inside a super conductor B = 0.

This Maxwell's equation fails to explain the electrodynamics of super conductor.

To rectify the problem London postulated the relation

$$\nabla^2 B=\frac{B}{\alpha}$$ ....(35)

ie $\nabla^2 B=\frac{\mu_0 n_s e^2}{m}B$ ....(36)

This equation is the second London's equation

The solution for the above equation is

$$B(x)=B_o\exp\left(\frac{-x}{\sqrt{\alpha}}\right)$$ ....(37)

ie $B(x)=B_o\exp\frac{-x}{\lambda}$ ....(38)

where $\lambda=\sqrt{\alpha}=\sqrt{\frac{m}{\mu_0 n_s e^2}}$ is called the penetration depth. Equation (38) shown that the magnetic field decreases exponentially as one proceeds from the surface to inside a super conductor. Then the mag. Field vanishes inside a super conductor in accordance with Meissner effect. But the equation predicts penetration of mag. Field inside a superconductor. The parameter $\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}}$ is called London penetration depth. This prediction was later experimentally verified.

The London penetration depth λ is found to vary with temperature

$$\lambda(T) = \lambda(o)\left[1 - \frac{T^4}{T_c^4}\right]^{-1/2}$$

Where $\lambda(o) = \sqrt{\frac{m}{\mu_0 n e^2}}$ is the penetration depth at OK. It is seen that λ increases as T increases and at T = Tc the substance becomes normal.

Also $J_2(x) = -J_s(o) e^{-\frac{x}{\lambda}}$

Thus the current also decays exponentially as one mores inside a super conductor. Thus Meissner effect is accompanied by a surface current and this surface current acts as a shield for the external magnetic field resulting in Meissner effect. Thus in a superconductor current flow through the surface.

NPTEL-(https://www.youtube.com/watch?v=RPusJNXEVMQ&list=PLbMVogVj5nJRjLrXp3kMtrIO8kZl1D1Jp&index=21).

Theory of super conductivity – BCS theory

The modern theory of superconductivity was put forward by Bardeen, Cooper and Schrieffer in 1957. This theory explains all the observed phenomena relating to superconductors. The theory was based on quantum treatment and can explain various effects such as zero resistance, Meissner effect etc.

The Salient features of the BCS theory are

1. An attractive interaction between electrons can lead to a ground state separated from excited states by an energy gap. The properties of superconductors such as critical field, electromagnetic and thermal properties etc result from the origin of the energy gap.

2. The electron lattice electron interaction leads to an energy gap. This indirect interaction between electron and electron takes place via lattice. When one electron interacts with lattice and deforms it. A second electron visualizes a

deformed lattice and adjoins it self to lower its energy. Thus the second electron interacts with first electron via the lattice.

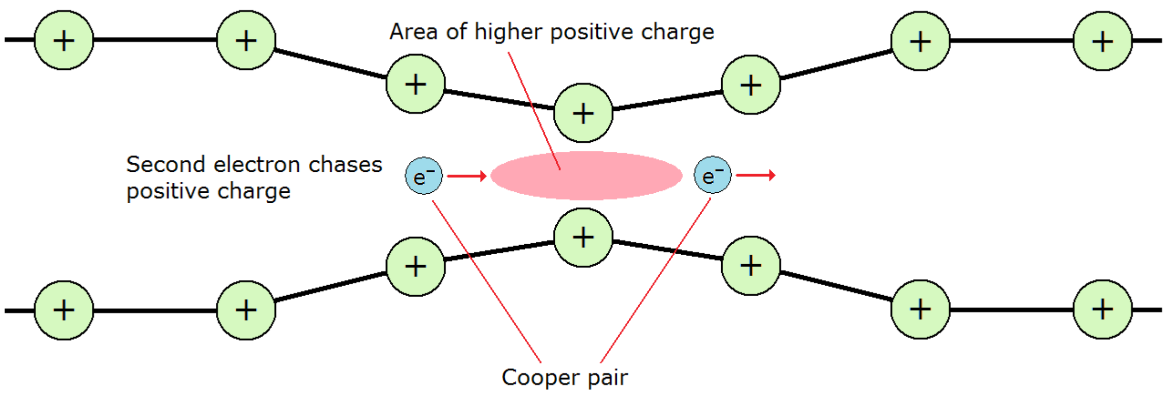

Fig-4

Suppose that two electrons (1) and (2) pass each other. The first electron attracts the +ve ions towards it self and second electron sees an electron which is screened by the ions. This screening reduces the effective charge of this electron and may result in a net positive charge. Thus electrons attract each other with the help of lattice. These pain is called a cooper pain. In this interaction the first electron emits a phonon which is absorbed by the second electron. Thus this interaction is called electron-phonon interaction.

As a result of this binding, an energy gap appears.

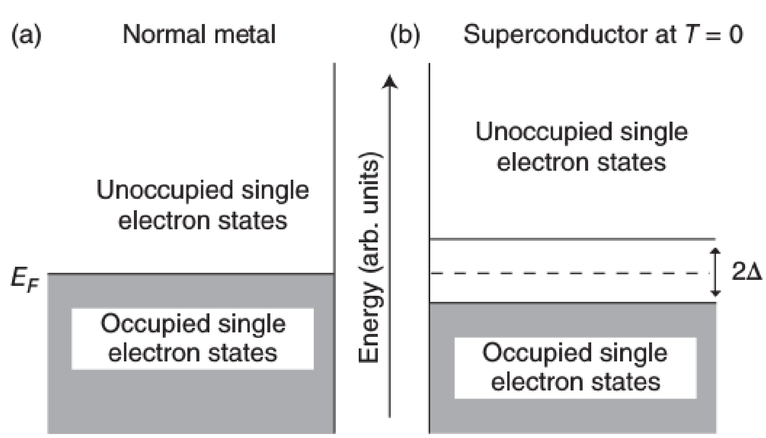

Fig-5

The energy gap can be shown to be equal to $\Delta_0 = 4\hbar\omega_D e^{\frac{-2}{g(E_F)}V'}$

Where W_D is the Debye frequency, $g(E_F)$ is the density of states of a normal metal at Fermi level and V' is the strength of electron lattice interaction.

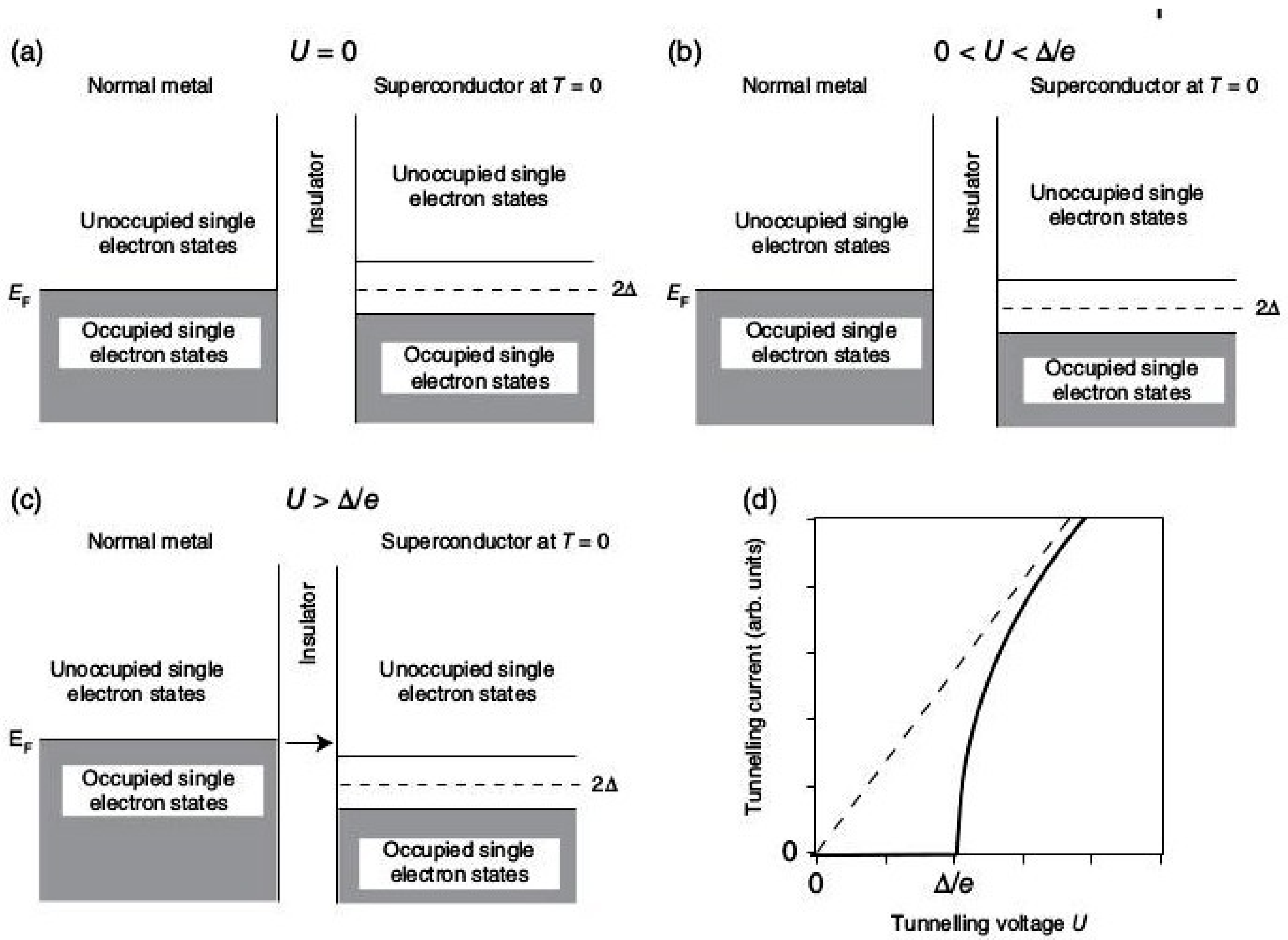

Figure 10.10 Tunneling experiment between a superconductor and a normal metal. The two are separated by a thin insulating oxide. Only elastic single-electron tunneling is considered. (a) Situation without applied voltage. (b) For a small applied voltage no tunneling is possible because of the lack of available states in the gap. (c) As the tunneling voltage exceeds Δ/e, single-electron tunneling becomes possible. (d) Thick line: tunneling current vs. voltage for the present junction; dashed line: corresponding curve for tunneling between two metals.

The value of Δ_0 is found to be 10^{-4} eV which is in agreement with observation.

3. It is seen that $W_D \alpha M^{-\frac{1}{2}}$ where M is the mass of ion. Therefore $\Delta_0 \alpha M^{-\frac{1}{2}}$. The energy gap and hence critical temperature decreases as M increases. This can be experimentally verified by substituting with isotopes. This properly ie $T_c \alpha M^{-\frac{1}{2}}$ is known as isotope effect. This provides the strongest evidence that lattice take part in super conductivity.

4. The penetration depth and coherence length emerge as consequences of BCS theory. The Meissner effect also emerges as a natural way.

5. It is also seen that the magnetic field through a superconducting ring is quantized and effective unit charge is found to be 2e. Thus BCS theory involves pairing of electrons.

NPTEL-(https://www.youtube.com/watch?v=qTzgjnwN2EU&t=7s).

Quantum Mechanical Tunneling and Josephson Effect

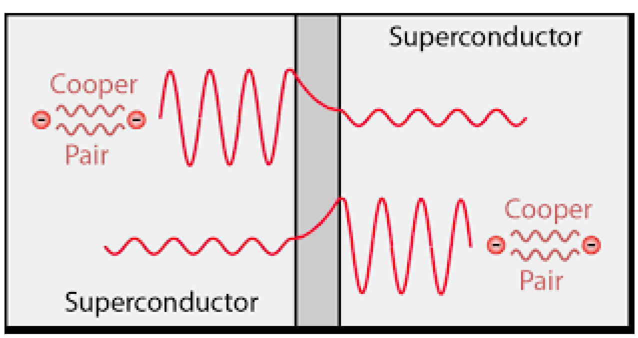

Fig-5

Consider two metals separated by an insulator. Normally insulator do not allow the flow of electrons from one metal to another. But if the barrier is very thin ie less than 20A° then there is a significant probability that an electron wheel impinges on the barrier will pass from one metal to another. This phenomena is called ‘tunneling’.

Under suitable condition remarkable effects are observed with tunding of super conducting pairs (Cooper pairs) from one super conductor to another super conductor separated by a thin layer if insulator. This tunneling happening in superconductors is termed as Josephson tunneling.

DC Josephson Effect

A dc current flows across the junction in the absence of any electric or magnetic field. The principle behind this effect is that the wave function on both sides would be highly correlated.

$$J = J_1 \sin \varphi_0$$

J1 is a measure of probability of transition across the function.

AC Josephson effect

A dc voltage applied across the function causes radio frequency oscellations across the function

$$J = J_1 \sin\left[\varphi_0 + \frac{2eV}{\hbar} t\right] \qquad \hbar = \frac{h}{2\pi}$$

Where V is the applied dc voltage. Thus application of static potential leads to an alternating current.

The angular frequency $w = \frac{2eV}{\hbar}$

But $\omega = 2\pi\upsilon$

Thus $\upsilon = \frac{eV}{\hbar\pi}$

Substituting for $e = 9.1\times10^{-19} c$ $\hbar = 1.06\times10^{-34} JS$ and $\pi = 3.14$

$$\upsilon = 484\,V\ GH_Z \qquad 1\ GH_z = 10^9\,Hz$$

V will be in the order of millivolts. These results were predicted by Josephson in 1962 which are in agreement with experimental results. A dc volt of IMV produces a frequency 483.6 MHz.

NPTEL-(https://www.youtube.com/watch?v=WETC7HgOgHo).

Type I and Type II Super Conductors

In some superconductors, application of a particular magnetic field destroys super conductivity abruptly. The transition from superconducting to normal state is sudden and abrupt as shown in figure. The value of applied field

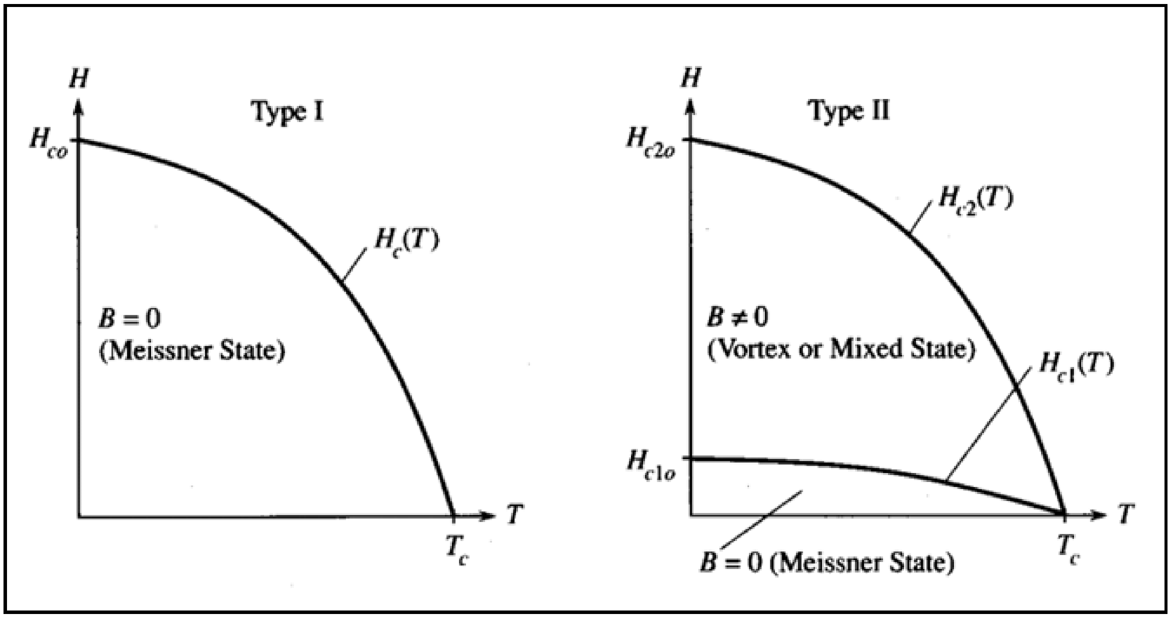

Fig-6

At which transition happens is called critical field. In type I Meissner effect is complete.

In type II Superconductors the transition from superconducting to normal state by applied magnetic field is not abrupt. The Meissner effect is only partial. Here the magnetic flow partially penetrates the super conductor. It starts at a value H_C, and completes at Hc_2. Up to H_C, the specimen is in super conducting state while between Hc_1 and Hc_2 the specimen is said to be in mixed state or vertex state. In the vertex state the specimen contains. Small circular normal regions surrounded by large superconducting regions. The small normal regions are referred to as vortex or fluxoids.

NPTEL-(https://www.youtube.com/watch?v=VHp2Ff5N_bs).

High Temperature Superconductivity

In High temperature Super conductors, the critical temperature at which super conductivity is manifested is high. Copper oxides such as YBaCaO compounds and certain ceramic compounds show high Tc value.

Applications of super conductors

1) In transmission of electricity from one place to another with out transmission loss.
2) Using Meissner effect we can have levitating trains.
3) Super conducting magnets with high magnetic intensity of several Tesla can be fabricated.
4) Josephson tunneling can be used for the accurate determination of $\hbar/e$
5) Quantum interference can be used in the construction of sensitive magneto meters.

SQUID (Super Conducting Quantum Interference Device)

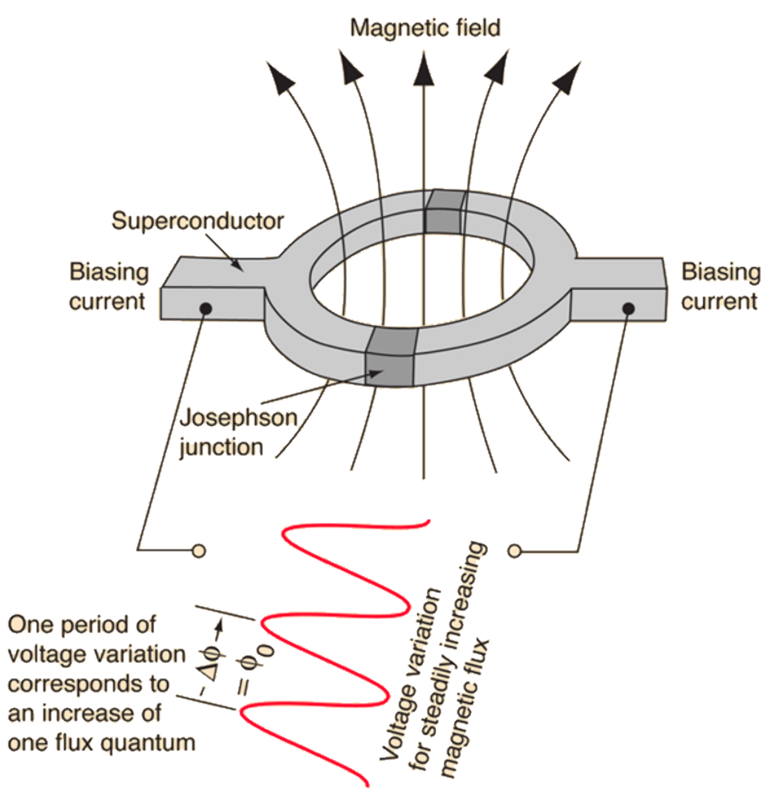

Fig-7

Consider two Josephson functions as shown a and b are insulators.

Let the phase difference through the path a in δ_a and that of path b is δ_b .

In the absence of applied field $\delta_a = \delta_b$

If we apply a flues φ through the cirinit then

$$\delta_a - \delta_b = \left(\frac{2e}{\hbar c}\right)\varphi$$

The current varies with φ and has maxima when

$$\frac{e\varphi}{\hbar c} = s\pi$$

where s is an integer.

This can be used to construct sensitive magnetometers squid is a very sensitive magnetometer used to measure extremely small magnetic fields based on superconducting loops containing Josephson junctions.

References:

1. Michael Tinkham – Introduction to Superconductivity (McGraw-Hill,Inc).
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3. Mit-Supercoductivity.(https://ocw.mit.edu/high-school/physics/exam-prep/electromagnetism/electromagnetic-induction/superconductivity/)
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